

B.Sc. Semester - 1 (NEP-2020) Examination

OCT/NOV-2024

MAT: Matrices and Coordinate Geometry (Minor)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any ONE out of TWO. (05)

- (1) Define: Periodic Matrix, Idempotent Matrix, Involutory Matrix, Unitary Matrix, Hermitian Matrix.
- (2) In usual notation prove that Matrix multiplication is Associative.

Que.1(B) Answer any ONE out of TWO. (05)

- (1) For $A=[a_{ij}]$ $n \times n$ Matrix, prove that $A(\text{adj } A) = |A| I$
- (2) Prove that every square Matrix can be expressed uniquely as the sum of a symmetric and skew symmetric Matrix.

Que.2(A) Answer any ONE out of TWO. (05)

- (1) Define Rank of Matrix using linearly independent rows or columns of

Matrix. Using this definition find Rank of Matrix $A = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$

- (2) Prove that the row rank of a Matrix is the same as its rank.

Que.2(B) Answer any ONE out of TWO. (05)

- (1) Define normal form of a matrix. Find Rank of Matrix $A = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ by represent it to normal form.

- (2) Prove that rank of product of two Matrices cannot exceed the rank of either Matrix.

Que.3(A) Answer any ONE out of TWO. (05)

- (1) State and prove Cayley- Hamilton theorem.
- (2) Prove that there is a unique eigen value with respect to any eigen vector of given matrix.

Que.3(B) Answer any ONE out of TWO. (05)

- (1) Show that the equation $AX=0$ has a nonzero solution iff A is singular.
- (2) Solve the linear equations: $x+y+z=9$, $2x+5y+7z=52$, $2x+y-z=0$.

Que.4(A) Answer any ONE out of TWO. (05)

- (1) Derive relation between Cartesian coordinates and polar coordinates. Also find Cartesian coordinates of $(2, 2\pi/3)$.
- (2) Derive formula to find distance between two points in polar coordinates. Also find the distance between $(a, \pi/2)$ and $(3a, \pi/6)$.

Que.4(B) Answer any ONE out of TWO. (05)

- (1) Obtain the equation of circle where centre is (p, α) and radius α .
- (2) Obtain the equation of line passing through $(1, \pi)$ and $(2, \pi/2)$.

Que.5(A) Answer any ONE out of TWO. (05)

- (1) In usual notation prove that $\text{adj}(AB) = \text{adj}(B) \text{adj}(A)$
- (2) If matrix $\begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find value of l, m, n .

Que.5(B) Answer any ONE out of TWO. (05)

- (1) Find eigen values and eigen vectors of $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
- (2) Find area of triangle with vertices $(-3, 30^\circ)$, $(5, 150^\circ)$, $(7, 210^\circ)$.
